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Stochastic Damped Free Vibration Analysis of Composite Sandwich Plates

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Abstract

In this paper, stochastic damped free vibration analysis of composite sandwich plates has been carried out using a nine node Heterosis plate bending element based on a first order shear deformation theory with a-priori shear correction factors. The plate bending element contains one transverse displacement and two rotations of the normals about the plate's mid-plane. Selective reduced integration scheme is adopted to integrate terms associated with the stiffness matrix formulations. Both lumped and consistent mass matrices are considered in the analysis. The accuracy and reliability of the present finite element formulation is verified with previously published results in the literature. New results are presented which are beneficial for designers of composite structures.

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Keywords:Stochastic Analysis; Heterosis Plate bending element; Damped Free vibration; Composite Sandwich plates; Lumped and Consistent Mass Matrices;

Nomenclature

a,b	in-plane dimensions of the plate along the x and y directions respectively

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1. Introduction

Composite materials with tailored properties are being increasingly used in load bearing applications due to various advantages. However, structural design complexities and experimental errors, unavoidable uncertainties in the design parameters of laminated composite sandwich structures may occur which are random in nature. Thus, safer and reliable predictions of composite sandwich structural dynamic response can be achieved via a probabilistic modelling and analysis in recent years. Singh and Grover (2013) [1] discussed about the involvement of various processes / parameters in the manufacturing and fabrication of laminated composites and the lack of control over these constituent processes causing the uncertainty in the system parameters. The various techniques available for the uncertainty characterization and their propagation in the deterministic framework are discussed. Mehrez et al (2010) [2] carried out a validation study of a stochastic representation of composite material properties from limited experimental data. The frequency response functions measured in these tests are implemented in a deterministic inverse problem in order to construct a database consisting of spatial estimations of the Young's modulus of the composite material. Vinckenroy and Wilde (1995) [3] used Monte Carlo techniques in statistical finite element methods for the determination of the behavior of composite materials structural components. Oh and Librescu (1997) [4] analyzed the problem of free vibration and reliability of cantilever composite plates featuring structural uncertainties. Mackerle (2002) [5] gave a bibliographical review of the finite element analyses of sandwich structures from theoretical and practical points of view on material and mechanical properties of sandwich structures, vibration, dynamic response and impact problems, heat transfer and thermo mechanical responses, contact problems, fracture mechanics, fatigue and damage and stability problems. Nayak (2013) [6] presented a detailed account of the finite element modeling of composite wind turbine blades. Finite elements for composite sandwich plates and shells are covered. Moreira and Rodrigues (2010) [7] proposed a simple and cost effective layer wise finite element model based on a two dimensional displacement field descriptor. Experiments and finite element analysis are carried out on free vibration response of soft core sandwich panels. Ru, Zhao and Zhu (2011) [8] carried out vibration analysis of Reissner-Mindlin isotropic Plates using quadrilateral Heterosis element.

From a literature review it has been found that the reliability damped free vibration analysis of composite sandwich panels is scantily covered. In this paper popular a nine node Heterosis quadrilateral plate bending element (Hughes and Cohen 1978 [9], Butalia et al (1990) [10]) with a-priori shear correction factors from shear strain energy formulations (Vlachoutsis 1992) [11] is used for the reliability of composite sandwich plates under damped free vibration condition. New results are presented which could serve benchmark for the designers of composite sandwich constructions.

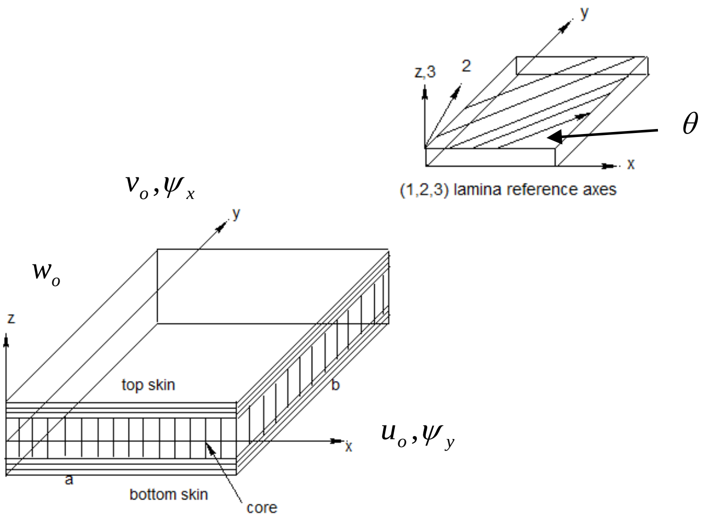

Fig. 1. Sandwich geometry with laminate orientations

2. Formulation of the Problem

The displacement field of the first-order shear deformation plate theory (Satapathy and Nayak (2014) [12]) (Fig. 1) is given as

$$u_\alpha = z\psi_\alpha \quad w = w^o \tag{1}$$

Where the superscript zero denotes the middle surface displacements; ψ_α are the rotations about the α axes; *h* is the thickness of the plate; Greek subscripts range on x and y; u_α are the displacements of a point in the α axes; and w is the displacement in the z direction, a and b are planar dimensions of the plate.
The strain-displacement relations are obtained from Eq. (1) which can be stated as

$$\varepsilon_{\alpha\beta} = z\kappa^o_{\alpha\beta} \quad \kappa^o_{\alpha\beta} = \frac{1}{2}\left(\psi^o_{\alpha,\beta} + \psi^o_{\beta,\alpha}\right) \quad \gamma^s_\alpha = 2\varepsilon_{\alpha\beta} = w_{,\alpha} + \psi_\alpha \tag{2}$$

Where a comma denotes partial differentiation and repeated indices imply summation.
The stress-strain relationships for the lamina in the laminate coordinates *(x,y,z)* are given by

$$\sigma_{xx} = \overline{Q}_{11}\varepsilon_{xx} + \overline{Q}_{12}\varepsilon_{yy} + 2\overline{Q}_{16}\varepsilon_{xy} \quad \sigma_{yy} = \overline{Q}_{12}\varepsilon_{xx} + \overline{Q}_{22}\varepsilon_{yy} + 2\overline{Q}_{26}\varepsilon_{xy} \quad \sigma_{xy} = \overline{Q}_{16}\varepsilon_{xx} + \overline{Q}_{26}\varepsilon_{yy} + 2\overline{Q}_{66}\varepsilon_{xy}$$
$$\sigma_{xz} = 2\overline{Q}_{55}\varepsilon_{xz} + 2\overline{Q}_{54}\varepsilon_{yz} \quad \sigma_{yz} = 2\overline{Q}_{45}\varepsilon_{xz} + 2\overline{Q}_{44}\varepsilon_{yz} \tag{3}$$

Where $\overline{Q}_{ij}$ are the transformed, plane stress reduced stiffness coefficients, which is given as,

$$\begin{gathered}
\overline{Q}_{11} = Q_{11}c^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}s^4 \quad \overline{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})c^2s^2 + Q_{12}(c^4 + s^4) \\
\overline{Q}_{16} = \left(Q_{11}c^2 + (Q_{12} + 2Q_{66})(s^2 - c^2) - Q_{22}s^2\right)cs \quad \overline{Q}_{22} = Q_{11}s^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}c^4 \\
\overline{Q}_{26} = \left(Q_{11}s^2 + (Q_{12} + 2Q_{66})(c^2 - s^2) - Q_{22}c^2\right)cs \quad \overline{Q}_{44} = Q_{44}c^2 + Q_{55}s^2 \quad \overline{Q}_{45} = (Q_{55} - Q_{44})cs \\
\overline{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12})c^2s^2 + Q_{66}(c^2 - s^2)^2 \quad \overline{Q}_{55} = Q_{55}c^2 + Q_{44}s^2
\end{gathered} \tag{4}$$

Where $\lfloor Q_{ij} \rfloor_k$ is the constitutive matrix at the lamina level; $c = \cos\theta$; $s = \sin\theta$; θ is the angle between the lamina x-axis and lamina principal $x_i$axis. The reduced stiffness components, Q_{ij}, are stated as

$$Q_{11} = \frac{E_{LL}{}^*}{1 - \nu_{LT}\nu_{TL}} \quad Q_{12} = Q_{21} = \frac{\nu_{LT}E_{TT}{}^*}{1 - \nu_{LT}\nu_{TL}} = \frac{\nu_{TL}E_{LL}{}^*}{1 - \nu_{LT}\nu_{TL}} \quad Q_{22} = \frac{E_{TT}{}^*}{1 - \nu_{LT}\nu_{TL}} \quad Q_{66} = G_{LT}{}^* \quad Q_{55} = G_{LZ}{}^* \quad Q_{44} = G_{TZ}{}^* \tag{5}$$

Where $E_{ii}{}^*(i = L, T)$ are the Young's moduli of the viscoelastic composite material, $G_{LT}{}^*$, $G_{LZ}{}^*$ and $G_{TZ}{}^*$ are the shear moduli of the composite material and ν_{LT} and ν_{TL} are the Poisson's ratios. The details on complex moduli of viscoelastic composites are given in the works of Nayak, Shenoi and Moy [14].
The governing equations of motion from Eqs. (1)-(5) can be stated as

$$\int_0^t\int_V \left(\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right)dVdt = \int_0^t\int_V \rho\left(\ddot{u}_x\delta u_x + \ddot{u}_y\delta u_y + \ddot{w}\delta w\right)dVdt \tag{6}$$

Where V is the volume, $\rho(x, y, z)$ is the density of the plate. Using Eqs. (1)-(5) in Eq. (6), the following equations of motion are obtained:

$$Q_{xz,x} + Q_{yz,y} = I_1 \ddot{w}^o \; M_{xx,x} + M_{xy,y} - Q_{xz} = I_3 \ddot{\psi}_x \; M_{xy,x} + M_{yy,y} - Q_{yz} = I_3 \ddot{\psi}_y \tag{7}$$

The various stress resultants are given by:

$$M = [D]\kappa^o_{\alpha\beta} \; Q = [A]^S \gamma^s_\alpha \tag{8}$$

Where

$$M = (M_{xx} \quad M_{yy} \quad M_{xy})^T \; Q = (Q_{xz} \quad Q_{yz})^T \; (D_{ij}) = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} (\overline{Q}_{ij})_k (z^2) dz (i, j = 1,2,6) \; (A^s_{ij}) = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} K^2_{ij} (\overline{Q}_{ij})_k dz (i, j = 5,4) \tag{9}$$

Where K^2_{ij} are the shear correction factors. In the present study, the shear correction factors are calculated on the basis of the transverse shear strain energy (Vlachoutsis, 1992 [11]). The inertias $I_i (i = 1,2,3)$ are defined by

$$(I_1, I_2, I_3) = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \rho_k (1, z, z^2) dz \tag{10}$$

Using Eqs. (7)-(10), the principle of virtual work equation can be obtained in the following form

$$\int_0^t \int_A \left(\delta\kappa^{o\,T}_{\alpha\beta} [D] \kappa^o_{\alpha\beta} + \delta\gamma^{s\,T}_\alpha [A^s] \gamma^s_\alpha \right) dAdt = \int_0^t \int_A \left[I_1 \ddot{w}^o \delta w^o + I_3 \left(\ddot{\psi}_x \delta\psi_x + \ddot{\psi}_y \delta\psi_y \right) \right] dAdT \tag{11}$$

3. Finite element approximation

A nine node Heterosis plate bending finite element is developed based on a first order shear deformation theory as discussed in the previous section. Each element 'e' has 'n' nodes, where each node i (i=1,…..,n) is identified with three degrees of freedom $U^i_{(e)} = \left(w^{oi}, \psi^i_x, \psi^i_y \right)_{(e)}$. The element displacement function approximations can be expressed as:

$$w^o = \sum_{i=1}^{n} N_i w^{oi} \, , \; \psi_x = \sum_{i=1}^{n} N_i \psi^i_x \, , \; \psi_y = \sum_{i=1}^{n} N_i \psi^i_y \tag{12}$$

Where $N_{i,} i = 1,.....,n$ are the interpolation functions. The shape functions N_i for an element are functions of the two reference variables ξ and η. Knowing the generalized displacement vector $\left(U_{(e)} = [N]^{(e)} \{\delta\}_{(e)} \right)$ at all points within the element 'e', the generalized mid-surface strains at any point in the element 'e' from Eq. (2) can be expressed in terms of nodal displacements as follows:

$$\kappa^{o(e)} = \left[B^o_\kappa \right]^{(e)} \{\delta\}_{(e)}, \quad \gamma^{s(e)} = \left[B^s_\varepsilon \right]^{(e)} \{\delta\}_{(e)} \tag{13}$$

Where $\left[B^o_\kappa \right]$ and $\left[B^s_\varepsilon \right]$ are the generated strain-displacement matrices.
The strain-displacement matrix in the generalized form is given as

$$\left[B^o_\kappa \right]^{(e)} = \begin{bmatrix} 0 & N_{1,x} & 0 & . & . & . & 0 & N_{n,x} & 0 \\ 0 & 0 & N_{1,y} & . & . & . & 0 & 0 & N_{n,y} \\ 0 & N_{1,y} & N_{1,x} & . & . & . & 0 & N_{n,y} & N_{n,x} \end{bmatrix} \left[B^s_\varepsilon \right]^{(e)} = \begin{bmatrix} N_{1,x} & N_1 & 0 & . & . & . & N_{n,x} & N_n & 0 \\ N_{1,y} & 0 & N_1 & . & . & . & N_{n,y} & 0 & N_n \end{bmatrix} \tag{14}$$

For arbitrary values of virtual displacements, the global dynamic equation can be formed from Eq. (11) as

$$[M]\{\ddot{\Delta}\}+[K]\{\Delta\}=0 \tag{15}$$

Here the unknown vector $\{\Delta\}$ is generated by the assemblage of element degrees of freedom $\{d\}_e^T, e=1,.....,$ total degrees of freedom in the region R. The assembled stiffness and mass matrices for free vibration analysis are

$$[K]=\sum_e \int_{A_e} \left[B_\kappa^{oT}[D]B_\kappa^o + B_\varepsilon^{sT}\left[A^s\right]B_\varepsilon^s\right] dA \tag{16}$$

$$[M]=\sum_e \int_{A_e} N^T M_I N dA \tag{17}$$

$[M_I]$ is the mass matrix containing inertia terms.

$$[M_I]=\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_3 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \tag{18}$$

The 3x3 Gauss-Legendre rule (i.e full integration scheme) is employed to integrate bending and inertia terms in the energy expressions for the Heterosis element. Whereas the 2x2 Gauss-Legendre rule (i.e reduced integration scheme) is employed to integrate shear terms in the energy expressions. Lobatto integration scheme (Islam and Saha 2008 [13]) is used to obtain the lumped mass matrix. The complex eigenvalue problem as given by Eq. (15) is solved by the complex Modulus (CM) method (Nayak et al. 2002 [14]).

4. Results and discussions

This section presents some numerical results using a nine node Heterosis finite element formulation (HT9) based on the first-order shear deformation theory.The first example compares the accuracy of free vibration results from the present formulation with an analytical solution (Meunier and Shenoi, 1999 [15]) and finite element analysis (FEA) solution from an assumed strain approach (Nayak, Shenoi and Moy, 2002 [14]) when applied to FRP sandwich plates made with PVC foam core, as shown in Fig. 2. The dimensional less natural frequencies $\varpi=\omega\left(a^2/h\right)\sqrt{\rho_c/E_c}$ are calculated for square sandwich plates with (0/90/0/core/0/90/0) layups, with the skins made of glass polyester resins and the core of HEREX C70.130 PVC foam. The material properties of unidirectional glass fibre in a polyester resin material are (Meunier and Shenoi, 1999 [15]): $E_1=24.51GPa,\ E_2=7.77GPa,$ $G_{12}=G_{13}=3.34GPa,\ G_{23}=1.34GPa,\ \rho_s=1800Kg/m^3,\ \nu_s=0.078$. The material properties of the HEREX C70.130 foam core product are (Meunier and Shenoi, 1999 [15]); $E_c=103.63MPa$, $G_c=50MPa,$ $\rho_c=130Kg/m^3,\ \nu_c=0.32$. A 6x6 mesh in full plate is used and the results are compared with analytical results (Meunier and Shenoi, 1999 [15]) and FEM results from HSDT (Nayak, Shenoi and Moy, 2002 [14]). Excellent agreement with the analytical and FEM results can be observed. Hence it can be concluded that the present first order theory with a-priori shear correction factors is also capable of modeling the sandwich plates in an accurate manner.

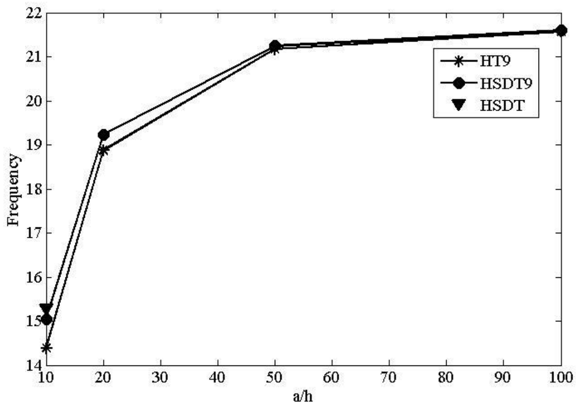

Fig. 2. Comparison of FEM models from the first order shear deformation theory (Heterosis FEM, HT9) and higher order shear deformation theory (Assumed strain FEM, HSDT9) and analytical solutions from Reddy's higher order shear deformation theory (HSDT)

Next example has been selected to compare the results of natural frequencies and loss factors of a square laminated composite plate with all edges free and with all eight 0^o layers from the present formulation with the analytical solution (Yarlagadda and Leisture 1995 [16]), the FEA results (Li, Ng and Adams 1984 [17]), the experimental results (Lin, Ni and Adams 1984 [17]) and the FEM results based on higher order theory (Nayak, Shenoi and Moy 2002 [14]). The material and geometrical properties of the composite plate are: a=b=0.178m, h=1.58x10^{-3} m. $E_1 = 172.7GPa$, $E_2 = 7.2GPa$ $G_{12} = G_{23} = G_{13} = 3.76GPa$, $\nu_{12} = 0.30$, $\rho = 1566kg/m^3$. The loss factors of the material are $\eta_1 = 0.0045$, $\eta_2 = 0.0422$, $\eta_1 = \eta_{23} = \eta_{13} = 0.0705$. A 10x10 mesh in the full plate is used and the results are tabulated in Table 1. The present results are in close agreement with the previously published results.

Table 1: Natural frequencies and loss factors of a square laminated plate with all eight 0^o layups.

Modes	Analytical (Yarlagadda and Leisture 1995 [16])	FEA (Lin, Di and Adams 1984 [17])	Expt (Lin, Di and Adams 1984 [17])	HSDT9 (Nayak, Shenoi and Moy 2002 [14])	Present (Consistent mass)	Present (Lumped)
			Natural frequencies f (Hz)			
1	81.56	83.57	81.50	81.71	81.3629	81.3628
2	110.53	118.42	107.40	109.93	109.7930	109.7873
3	202.08	207.79	196.60	200.37	199.4572	199.4643
			Loss factor $\eta(\%)$			
1	6.88	6.76	7.0	6.88	6.90	6.898
2	4.22	4.28	4.9	4.33	4.33	4.334
3	6.07	5.89	5.4	6.10	6.11	6.108

Next example considers the first order and higher order FE analysis of sandwich plate with skins made of carbon FRP plates with viscoelastic properties (HMX/DX-210) (Nayak, Shenoi and Moy 2002 [14]) and cores made of aluminum. The following material properties are considered: for face layers: $E_1 = 172.7GPa$, $E_2 = 7.2GPa$

$G_{12} = G_{23} = G_{13} = 3.76GPa$, $\nu_{12} = 0.30$, $\rho = 1566kg/m^3$. The loss factors of the face layer materials are $\eta_1 = 0.0045$, $\eta_2 = 0.0422$, $\eta_1 = \eta_{23} = \eta_{13} = 0.0705$. For Aluminum Core Crawley (1979) [18]: $E = 68.9GPa$, $\nu = 0.30$, $\rho_c = 2770Kg/m^3$. The following geometrical properties are adopted: a=0.152m, b=0.076m, ply thickness=0.00013m, core thickness=0.0010m Figs. 3 and 4 show the frequencies and loss factors for the first five modes for cantilever and clamped boundary conditions respectively by using HT9 and HSDT9. The results from both first order heterosis FEM model HT9 and higher order assumed strain FEM models HSDT9 (Nayak, Shenoi and Moy 2002 [14]) show close agreement with each other. From the above discussions, it can be concluded that the desired damping in a structure can be obtained by using different boundary conditions.

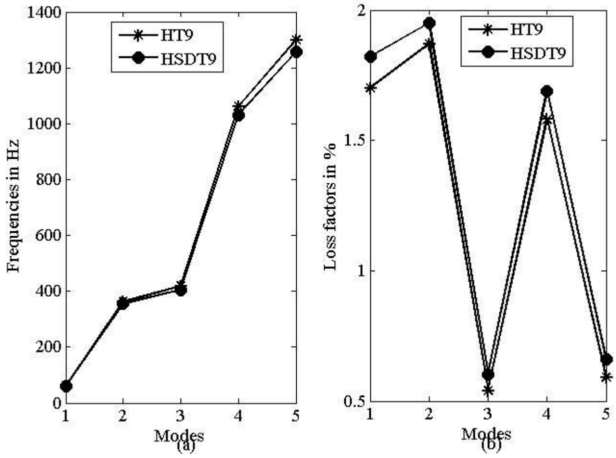

Fig. 3 Comparison of results from HT9 and HSDT9 for damped cantilever plates

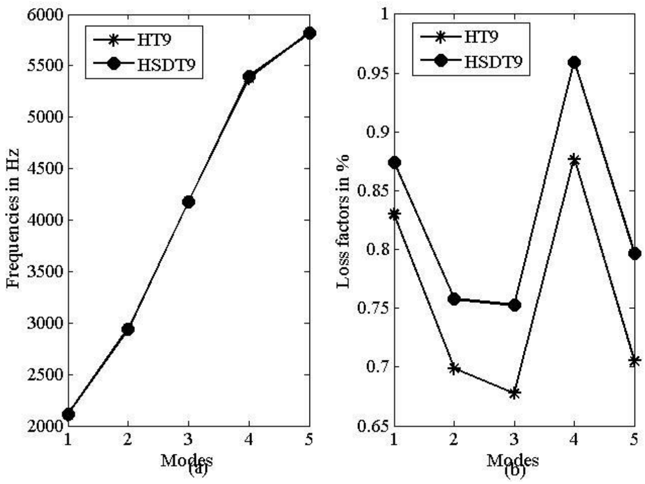

Fig. 4 Comparison of results from HT9 and HSDT9 for damped clamped plates

In the next example, the damped free vibration reliability analysis is carried out on a cantilever sandwich plate $\left(\pm 45/\mp 45/Core\right)_s$. The following material and geometrical properties are adopted: a=0.152m, b=0.076m, ply thickness=0.00013m, core thickness=0.0010m face layers: $E_1 = 172.7GPa$, $E_2 = 7.2GPa$ $G_{12} = G_{23} = G_{13} = 3.76GPa$, $\nu_{12} = 0.30$, $\rho = 1530kg/m^3$ (assumed). The loss factors of the face layer materials are $\eta_1 = 0.0045$, $\eta_2 = 0.0422$, $\eta_1 = \eta_{23} = \eta_{13} = 0.0705$. For Aluminum Core: $E = 68.9GPa$, $\nu = 0.30$, $\rho_c = 2770Kg/m^3$. A mesh density of 8x4 elements in a full plate model is taken.

The reliability of cantilevered composite sandwich $\left(\pm 45/\mp 45/Core\right)_s$ plate under an external oscillatory load featuring random moduli and Poisson's ratios is considered. Monte Carlo simulation method with 100 samples is adopted presently. The coefficients of variation (COV) for E, G and ν for the face and core layers are taken as 0.05 (graphite composite) and 0.10 (aluminum core) respectively in the analysis. The reliability results are displayed in Fig. 5 a and b. The fundamental natural frequency f_{rn} (Hz) of the composite sandwich plates with material uncertainties is normalized with the fundamental frequency f_{dn} and loss factor of the composite sandwich plates with deterministic properties. The structure is subjected to oscillatory load whose frequency f_o lies within the range of 0- f_{dn}. The structure is safe when its fundamental natural frequency is beyond f_o. From the results it is observed that the randomness in material properties in face and core layers contributes towards the variation of the natural frequencies and loss factors.

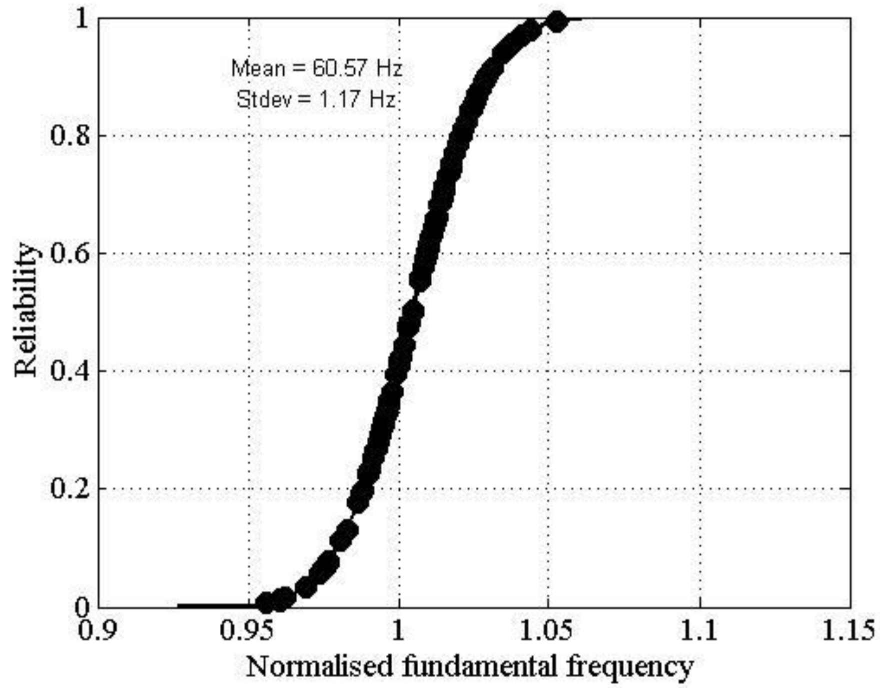

Fig 5a Reliability (fundamental frequency) of cantilevered composite sandwich $\left(\pm 45/\mp 45/Core\right)_s$ plate, under an external oscillatory load. The uncertainties concern the elastic moduli, shear moduli and major Poisson's ratio of both face and core.

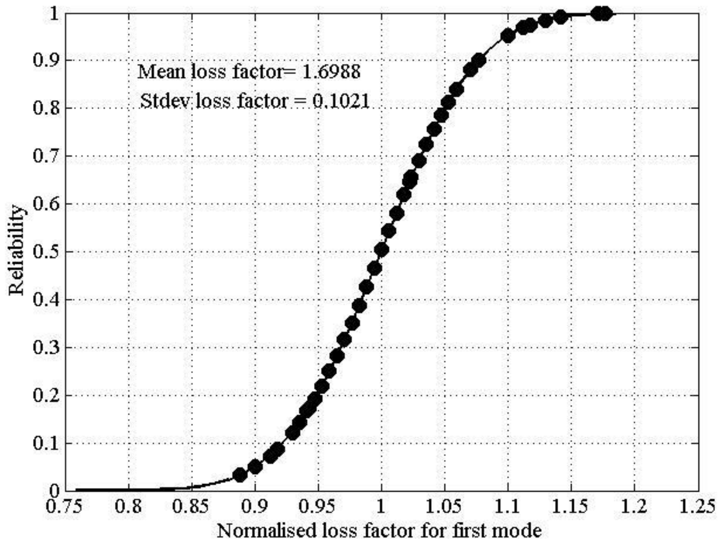

Fig 5b Reliability (loss factor for first mode) of cantilevered composite sandwich $\left(\pm 45/\mp 45/Core\right)_s$ plate, under an external oscillatory load. The uncertainties concern the elastic moduli, shear moduli and major Poisson's ratio of both face and core.

5. Concluding remarks

In this paper a nine node Heterosis finite element formulation is incorporated to deal with reliability damped free vibration of laminate and composite sandwich plates in a unified formulation. Both consistent and lumped mass matrices are used presently. The present results are compared with previously published experimental and numerical results. New results are presented on reliability damped free vibration of composite sandwich plates which could help designers of composite structures.

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